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The theory of judgment aggregation: an introductory review

Working paper

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Abstract

This paper provides an introductory review of the theory of judgment aggregation. It introduces the paradoxes of majority voting that originally motivated the field, explains several key results on the impossibility of propositionwise judgment aggregation, presents a pedagogical proof of one of those results, discusses escape routes from the impossibility and relates judgment aggregation to some other salient aggregation problems, such as preference aggregation, abstract aggregation and probability aggregation. The present illustrative rather than exhaustive review is intended to give readers new to the field of judgment aggregation a sense of this rapidly growing research area.

1 Introduction

The theory of judgment aggregation is a growing interdisciplinary research area in economics, philosophy, political science, law and computer science. Its main research question is the following: How can a group of individuals make consistent collective judgments on a set of propositions on the basis of the group members' individual judgments on them? This problem lies at the heart of democratic decision making and arises in many different contexts, ranging from legislative committees to referenda, from expert panels to juries and multi-member courts, from boards of companies to international organizations, from families and informal social groups to societies at large. While each such real-world case can be investigated in its own right, the theory of judgment aggregation looks at the structural properties that different judgment aggregation problems have in common, abstracting from the details of individual cases. The aim of this paper is to provide an introductory review of this theory.

The recent interest in judgment aggregation was sparked by the observation that majority voting, the paradigmatic democratic aggregation rule, fails to guarantee consistent collective judments whenever the decision problem in question exceeds a certain

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level of complexity. This observation, now known as the *discursive dilemma*, generalizes Condorcet's classic paradox of voting from the 18th century. The problem of inconsistent majority judgments was subsequently shown to illustrate a deeper impossibility result, of which the literature now contains several variants. The bottom line is that, for a large class of decision problems, no aggregation rule guarantees consistent collective judgments *and* satisfies some other salient conditions exemplified by majority voting, most importantly the feature of propositionwise aggregation, i.e., determining the collective judgment on each proposition as a function of individual judgments on it. The different variants of this impossibility result, in turn, allow us to identify how far we need to deviate from majority voting, and thereby to modify conventional democratic principles, in order to find workable solutions to judgment aggregation problems.

The theory of judgment aggregation has several different intellectual origins.[1] The initial observation that motivated much of the current field had its origins in the area of jurisprudence, in Kornhauser and Sager's work on decision making in collegial courts (Kornhauser and Sager 1986, 1993; Kornhauser 1992), but was given its present interpretation – as a general problem of inconsistent majority judgments – by Pettit (2001), Brennan (2001) and List and Pettit (2002). List and Pettit (2002, 2004) introduced a first formal model of judgment aggregation, combining an axiomatic approach to the study of aggregation rules, as common in social choice theory (going back to Arrow 1951/1963), with a logical representation of propositions. Using this model, they proved a simple impossibility theorem, which was strengthened and extended by several authors, beginning with Pauly and van Hees (2006) and Dietrich (2006). Independently, Nehring and Puppe (2002) proved some powerful results on the theory of strategy-proof social choice, which turned out to have important corollaries for judgment aggregation (see Nehring and Puppe forthcoming). Their key innovation was to characterize classes of decision problems for which certain impossibility results hold, which inspired a sequence of subsequent results, beginning with Dokow and Holzman (forthcoming), Dietrich and List (2007a) and others. A very general extension of the model of judgment aggregation, from propositional logic to any logic within a large class, was developed by Dietrich (2007a). The theory of judgment aggregation is also related to the theories of abstract aggregation (Wilson 1975, Rubinstein and Fishburn 1986), belief merging in computer science (Konieczny and Pino Pérez 2002, see also Pigozzi 2006) and probability aggregation (e.g., McConway 1981, Genest and Zidek 1986, Mongin 1995), and has informal precursors in the work of Guilbaud (1966) on what he called the logical problem of aggregation and arguably in Condorcet's work itself. The relationship between preference aggregation in the tradition of Condorcet and Arrow and judgment aggregation is discussed in List and Pettit (2004) and Dietrich and List (2007a).

This paper is structured as follows. In section 2, I explain the problem of inconsistent majority judgments, which sparked the interest in judgment aggregation. In section 3, I introduce the basic formal model of judgment aggregation, which then, in section 4, allows me to present some key results on the impossibility of propositionwise judgment aggregation. In section 5, I ask how this impossibility can be avoided, reviewing several possible escape routes. In section 6, I relate the theory of judgment aggregation to other branches of aggregation theory. In section 7, finally, I make some concluding remarks. While a comprehensive survey of the theory of judgment aggregation is beyond the scope

[1]For further technical and philosophical surveys, see List and Puppe (2009) and List (2006), respectively.

of this paper, the present illustrative rather than exhaustive review of the field is intended to introduce the theory in a succinct way, in light of the fact that the literature is now so large that readers new to the area may find it hard to know where to start.

2 The problem of inconsistent majority judgments

It is instructive to begin with Kornhauser and Sager's (1986) original example from the area of jurisprudence: the *doctrinal paradox* (the name was introduced in Kornhauser 1992). Suppose a collegial court consisting of three judges has to reach a verdict in a breach-of-contract case. There are three propositions on which the court is required to make judgments:

p: The defendant was contractually obliged not to do a particular action.
q: The defendant did that action.
r: The defendant is liable for breach of contract.

According to legal doctrine, propositions p and q are jointly necessary and sufficient for proposition r. Suppose now that the three judges disagree about the case, as shown in Table 1. The first thinks that p and q are both true, and hence that r is true as well. The second thinks that p is true but q is false, and consequently that r is also false. The third thinks that while q is true, p is false, and so r is false too. What does the court as a whole think?

	p	q	r
Judge 1	True	True	True
Judge 2	True	False	False
Judge 3	False	True	False
Majority	True	True	False

Table 1: A doctrinal paradox

If the three judges take a majority vote on proposition r – the *conclusion* – the outcome is its rejection: a 'not liable' verdict. But if they take majority votes on each of p and q instead – the *premises* – then both of these propositions are accepted and hence the background legal doctrine dictates that r should be accepted as well: a 'liable' verdict. The court's decision thus depends on the aggregation rule used. Under the first of these two approaches, the *conclusion-based rule*, the court will reach a 'not liable' verdict; under the second, the *premise-based rule*, it will reach a 'liable' verdict. Kornhauser and Sager's *doctrinal paradox* consists in the fact that the premise-based and conclusion-based rules can produce opposite outcomes for the same combination of individual judgments.[2]

However, the example also illustrates a more general point. Relative to the given legal doctrine – which states that r is true if and only if both p and q are true – the majority judgments across the three propositions are inconsistent. In precise terms, the set of propositions accepted by a majority, namely $\{p, q, \neg r\}$, is logically inconsistent relative to the constraint $r \leftrightarrow (p \wedge q)$. This problem generalizes well beyond this example

[2] For recent discussions of the doctrinal paradox, see Kornhauser and Sager (2004) and List and Pettit (2005).

and does not depend on the presence of any legal doctrine or other exogenous constraint; nor does it depend on any criterion for partitioning the relevant set of propositions into premises and conclusions.

To illustrate the more general problem, consider any set of propositions with some non-trivial logical connections; below I say more about the precise kinds of logical connections required. Take, for instance, the following three propositions on which a multi-member government may seek to make collective judgments:

p:	We can afford a budget deficit.
$p \rightarrow q$:	If we can afford a budget deficit, then we should increase spending on health care.
q:	We should increase spending on health care.

Suppose now that individual judgments are as shown in Table 2. One third of the individuals accept all three propositions; a second third accept p but reject $p \rightarrow q$ and q, accepting $\neg(p \rightarrow q)$ and $\neg q$ instead; and the last third accept $p \rightarrow q$ but reject p and q, accepting $\neg p$ and $\neg q$ instead.

	p	$p \rightarrow q$	q
1/3 of individuals	True	True	True
1/3 of individuals	True	False	False
1/3 of individuals	False	True	False
Majority	True	True	False

Table 2: A problem of inconsistent majority judgments

Then each individual holds consistent judgments on the three propositions, and yet there are majorities for p, for $p \rightarrow q$ and for $\neg q$, a logically inconsistent set of propositions. The fact that majority voting may generate inconsistent collective judgments is often called the *discursive dilemma* (Pettit 2001, List and Pettit 2002; see also Brennan 2001). The problem is very general:

> **Remark 1.** Inconsistent majority judgments can arise as soon as the set of propositions and their negations on which judgments are to be made exhibits a simple combinatorial property (Dietrich and List 2007b, Nehring and Puppe 2007): it has a minimally inconsistent subset of three or more propositions, where a set of propositions is called *minimally inconsistent* if it is inconsistent and every proper subset of it is consistent.

In the court example, a minimally inconsistent set with these properties is $\{p, q, \neg r\}$, where the inconsistency is relative to the constraint $r \leftrightarrow (p \wedge q)$. In the government example, it is $\{p, p \rightarrow q, \neg q\}$. It is easy to see that, as soon as there exists at least one minimally inconsistent subset of three or more propositions among the proposition-negation pairs under consideration, one can always construct combinations of judgments such as the one in Table 2 and hence arrive at inconsistent majority judgments. As explained in section 6 below, Condorcet's classic paradox of cyclical majority preferences is an instance of this general phenomenon, which Guilbaud (1952) described as the *Condorcet effect*.

3 The logic-based model of judgment aggregation

In order to go beyond the observation that majority voting may yield inconsistent collective judgments and to ask whether other aggregation rules may be immune to this problem, it is necessary to introduce a more general model, which abstracts from the specific decision problem and aggregation rule in question. The model to be presented follows the formalism introduced in List and Pettit (2002) and extended beyond standard propositional logic by Dietrich (2007a).

Consider a finite set $N = \{1, 2, ..., n\}$ (with $n \geq 2$) of individuals, who have to make judgments on some propositions.[3] Propositions are represented by sentences from propositional logic or a more expressive logical language and are denoted p, q, r and so on. Propositional logic can express atomic propositions, which do not contain any logical connectives, such as the proposition that we can afford a budget deficit or the proposition that spending on health care should be increased, as well as compound propositions, with the logical connectives $\neg$ ('not'), $\wedge$ ('and'), $\vee$ ('or'), $\rightarrow$ ('if-then'), $\leftrightarrow$ ('if and only if'), such as the proposition that *if* we can afford a budget deficit, *then* spending on health care should be increased. Instead of propositional logic, any logic with some minimal properties can be used, including expressively richer logics such as predicate, modal, deontic and conditional logics (Dietrich 2007a). Generally, a logic for the present purposes is a non-empty set $\mathbf{L}$ of sentences (called *propositions*) that is endowed with a *negation operator* $\neg$ ('not') and a notion of *consistency*, subject to some standard conditions.[4] In standard propositional logic, a set of propositions is *consistent* if all its members can be simultaneously true. Thus the set$\{p, q, p \wedge q\}$ is consistent while the sets $\{p, p \rightarrow q, \neg q\}$ and $\{p, \neg p\}$ are not. We say that a set $S \subseteq \mathbf{L}$ *logically entails* a proposition $p \in \mathbf{L}$, written $S \vdash p$, if $S \cup \{\neg p\}$ is inconsistent. The set $\{p, q\}$, for example, logically entails the proposition $p \wedge q$.

The set of propositions on which judgments are to be made in a particular decision problem is called the *agenda*. Formally, the *agenda* is defined as a non-empty subset $X \subseteq \mathbf{L}$ that is closed under negation, i.e., if $p \in X$, then $\neg p \in X$.[5] In the government example, the agenda is

$$X = \{p, \neg p, p \rightarrow q, \neg(p \rightarrow q), q, \neg q\}.$$

In the court example, it is

$$X = \{p, \neg p, q, \neg q, r, \neg r\},$$

but here there is an additional stipulation built into the logic requiring that $r \leftrightarrow (p \wedge q)$.[6]

[3] The agenda characterization results discussed below require $n \geq 3$.

[4] Every proposition-negation pair $\{p, \neg p\} \subseteq \mathbf{L}$ is inconsistent. Subsets of consistent sets $S \subseteq \mathbf{L}$ are consistent. The empty set $\emptyset$ is consistent, and every consistent set $S \subseteq \mathbf{L}$ has a consistent superset $T \subseteq \mathbf{L}$ containing a member of each proposition-negation pair $\{p, \neg p\} \subseteq \mathbf{L}$. See Dietrich (2007a).

[5] For some formal results, it is necessary to exclude tautologies and contradictions from the agenda, i.e., to assume that each of $\{\neg p\}$ and $\{p\}$ is consistent for every $p \in X$. Further, some results simplify when the agenda is assumed to be finite. In order to avoid such technicalities, I here make both simplifying assumptions. To render finiteness compatible with negation-closure, I assume that double negations cancel each other out; more elaborate constructions can be given.

[6] Formally, a set of propositions S is deemed to be consistent in this augmented logic if and only if $S \cup \{r \leftrightarrow (p \wedge q)\}$ is consistent in the standard sense of propositional logic. For the full details of this construction, see Dietrich and List (2008d).

Now each individual's *judgment set* is the set of propositions that this individual accepts. Formally, a *judgment set* is a subset $J \subseteq X$. On the standard interpretation, to accept a proposition means to believe it to be true; on an alternative interpretation, it could mean to desire it to be true. For the present purposes, it is easiest to adopt the standard interpretation, i.e., to interpret judgments as binary cognitive attitudes rather than as binary emotive ones. A judgment set is called *consistent* if it is a consistent set of propositions in the standard sense of the logic, and *complete* if it contains a member of each proposition-negation pair in X. A combination of judgment sets across the individuals in N, $(J_1, ..., J_n)$ is called a *profile.* Thus the first three rows of Tables 1 and 2 show examples of profiles on the agendas in question.

To complete the exposition of the model, it remains to define the notion of an aggregation rule. An *aggregation rule* is a function F that maps each profile of individual judgment sets $(J_1, ..., J_n)$ in some domain to a collective judgment set $J = F(J_1, ..., J_n)$, interpreted as the set of propositions accepted by the group N as a whole. Examples of aggregation rules are:

- *majority voting*, as already introduced, where each proposition is collectively accepted if and only if it is accepted by a majority of individuals, i.e.,

$$F(J_1, ..., J_n) = \{p \in X : |\{i \in N : p \in J_i\}| > n/2\};$$

- *supermajority* or *unanimity rules*, where each proposition is collectively accepted if and only if it is accepted by a certain qualified majority of individuals, such as two thirds, three quarters, or all of them, i.e.,

$$F(J_1, ..., J_n) = \{p \in X : |\{i \in N : p \in J_i\}| \geq nq\}, \text{ where } q \in (\frac{1}{2}, 1];$$

- *dictatorships*, where the collective judgment set is always the individual judgment set of some antecedently fixed individual, i.e.,

$$F(J_1, ..., J_n) = J_i \text{ for some fixed } i \in N;$$

- *inverse dictatorships*, where the collective judgment set is always – rather perversely – the propositionwise negation of the individual judgment set of some antecedently fixed individual, i.e.,

$$F(J_1, ..., J_n) = \{\neg p : p \in J_i) \text{ for some fixed } i \in N;$$

- *premise-based rules*, as briefly introduced in the court example above, where the set of collectively accepted propositions is given by applying majority voting (or some other propositionwise criterion) on some privileged subset of propositions (the *premises*) and then taking the logical closure of the resulting judgments, e.g.,

$$F(J_1, ..., J_n) = \{p \in X : G(J_1, ..., J_n) \cap Y \vdash p\},$$

  where $G(J_1, ..., J_n) \cap Y$ is the set of majority-accepted propositions (or those picked out by another designated criterion G) among the ones in the privileged subset $Y \subseteq X$;

- *conclusion-based rules*, also mentioned above, where the set of collectively accepted propositions is given by applying majority voting (or some other criterion) on some privileged subset of propositions (the *conclusions*) but not taking the logical closure of the resulting judgments, e.g.,

$$F(J_1, ..., J_n) = G(J_1, ..., J_n),$$

with $G(J_1, ..., J_n)$ as just defined.

Although there is an abundance of logically possible aggregation rules, it is surprisingly difficult to find compelling such rules that guarantee consistent collective judgments. As we have already seen, majority voting notoriously fails to do the job as soon as the propositions in X exhibit some relatively simple logical connections (i.e., X has a minimally inconsistent subset of three or more propositions). Let me therefore turn to a more general, axiomatic investigation of possible aggregation rules.

4 The impossibility of propositionwise aggregation

Are there any democratically plausible aggregation rules that guarantee consistent collective judgments? The answer to this question depends on two factors:

- the types of agendas for which we want to employ such an aggregation rule; and
- the conditions that we expect the aggregation rule to satisfy.

Before presenting some illustrative results, let me briefly explain in very simple terms why both factors matter. Suppose, for example, we wish to make only a single binary judgment, say on whether to accept p or $\neg p$, i.e., the agenda contains only a single proposition-negation pair (or perhaps multiple unconnected such pairs). Obviously, we can then use majority voting without the risk of collective inconsistency. On the other hand, if the agenda has a minimally inconsistent subset of three or more propositions, majority voting runs into difficulties, as illustrated above. These simple considerations highlight that the complexity of the agenda is of crucial relevance to the question of which aggregation rules, if any, produce consistent collective judgments.

Secondly, suppose that, instead of using an aggregation rule that satisfies strong democratic principles, we content ourselves with installing a dictatorship, i.e., we appoint one individual whose judgments are deemed always to determine the collective ones. If this individual's judgments are consistent, then, trivially, so are the resulting collective judgments. The problem of aggregation will have been resolved under such a dictatorial arrangement, albeit in a degenerate and unappealing way. By contrast, if we demand democratic responsiveness of collective judgments to individual ones and interpret this in terms of majority voting, we can run into problems, as we have already seen. This illustrates that the answer to the question of whether there exist any aggregation rules that ensure consistent collective judgments depends very much on the conditions we expect those rules to meet.

4.1 A simple impossibility

With these preliminary remarks in place, we are in a position to address the question of the existence of compelling aggregation rules in more detail. The original impossibility theorem in List and Pettit (2002) gives a simple answer to this question for a specific class of agendas and a specific set of conditions on the aggregation rule. The agendas in question are those that contain at least two distinct atomic propositions (say, p, q) and either their conjunction ($p \wedge q$), or their disjunction ($p \vee q$), or their material implication ($p \rightarrow q$). There are four conditions on the aggregation rule: one *input condition*, one *output condition* and two *responsiveness conditions*. The first condition requires the aggregation rule to accept as admissible input every possible profile of fully rational individual judgment sets.

> **Universal domain.** The domain of F is the set of all possible profiles of consistent and complete ('fully rational') individual judgment sets on X.

The second condition constrains the outputs of the aggregation rule, requiring it to produce a fully rational collective judgment set for every admissible profile of individual judgment sets.

> **Collective rationality.** For any profile $(J_1, ..., J_n)$ in the domain of F, $F(J_1, ..., J_n)$ is a consistent and complete collective judgment set on X.

The third and fourth conditions constrain the way the outputs are generated from the inputs and can thus be seen as *responsiveness conditions*. We begin with systematicity.

> **Systematicity.** For any two profiles $(J_1, ..., J_n)$, $(J'_1, ..., J'_n)$ in the domain of F and any two propositions $p, q \in X$,
>
> $$[p \in J_i \Leftrightarrow q \in J'_i \text{ for all } i \in N] \Rightarrow [p \in F(J_1, ..., J_n) \Leftrightarrow q \in F(J'_1, ..., J'_n)].$$

Informally, this can also be expressed as the requirement that (i) the collective judgment on each proposition in X depend only on individual judgments on that proposition, not on individual judgments on other propositions (the *independence* part), and (ii) the criterion for determining the collective judgment on each proposition be the same across all propositions in X (the *neutrality* part). The next responsiveness condition requires that all individuals be given equal weight in the aggregation.

> **Anonymity.** For any two profiles $(J_1, ..., J_n)$, $(J'_1, ..., J'_n)$ in the domain of F which are permutations of each other, $F(J_1, ..., J_n) = F(J'_1, ..., J'_n)$.

Much can be said about these conditions – I discuss them further in the section on how to avoid the impossibility – but for the moment it is enough to note that they are inspired by key properties of majority voting. One can easily check that majority voting satisfies all of them, with the crucial exception of the consistency part of collective rationality (except for trivial agendas), as shown by the discursive dilemma. The following theorem establishes that majority voting is not alone in failing to satisfy the four conditions together.

> **Theorem 1.** Let $X \supseteq \{p, q, p \wedge q\}$ (where $p \wedge q$ can also be replaced by $p \vee q$ or $p \rightarrow q$). Then there exists no aggregation rule satisfying universal domain, collective rationality, systematicity and anonymity (List and Pettit 2002).

The proof is fairly simple, but is omitted here due to space constraints; interested readers are referred to the original paper.

4.2 The impossibility generalized

As mentioned in the introduction, this impossibility result has been significantly generalized and extended in a growing literature. Different impossibility theorems apply to different classes of agendas, and they impose different conditions on the aggregation rule. However:

> **Remark 2.** Different impossibility theorems share a generic form, which can be summarized roughly as follows: For a particular class of agendas, the aggregation rules satisfying a particular combination of input, output and responsiveness conditions are either non-existent or otherwise degenerate.

The precise class of agendas and input, output and responsiveness conditions vary from result to result. For example, Pauly and van Hees's (2006) first theorem states that if we take the same class of agendas as in List and Pettit's theorem and the same input and output conditions (universal domain and collective rationality), keep the responsiveness condition of systematicity but drop anonymity, then we are left only with dictatorial aggregation rules, as defined above.

Other theorems by Pauly and van Hees (2006) and Dietrich (2006) show that, for more restrictive classes of agendas (so-called *atomically closed* and *atomic* ones, respectively), again with the original input and output conditions and without anonymity, but this time with systematicity weakened to its first part (independence), we are still left only with dictatorial or constant aggregation rules. The latter are another kind of degenerate rules, which assign to every profile the same fixed collective judgment set, paying no attention to any of the individual judgment sets.

Another theorem, by Mongin (2008), also keeps the original input and output conditions, adds a further responsiveness condition requiring that any proposition $p \in X$ accepted by *all* individuals be collectively accepted, but weakens systematicity further, namely to an independence condition restricted to atomic propositions alone. The theorem then shows that, for a certain class of agendas, only dictatorial aggregation rules satisfy these conditions together. Later I also comment on some impossibility results that modify the input and output conditions introduced above.

4.3 Characterizations of impossibility agendas

The most general theorems in the literature on judgment aggregation are what we may call *agenda characterization theorems*:

> **Remark 3.** Agenda characterization theorems do not merely show that for a certain class of agendas, a certain combination of input, output and responsiveness conditions leads to an empty or degenerate class of aggregation rules, but they fully characterize those agendas for which this is the case and, by implication, those for which it is not.

The underlying idea was introduced by Nehring and Puppe (2002) in the different context of strategy-proof social choice, but several of their results carry over to judgment aggregation, as discussed in Nehring and Puppe (forthcoming), and have inspired a sequence of subsequent agenda characterization results (e.g., Dokow and Holzman forthcoming, Dietrich and List 2007a).

To give a flavour of these results, recall that only agendas with at least one minimally inconsistent subset of three or more propositions are of interest from the perspective of impossibility theorems; call such agendas *non-simple*. For agendas below this level of complexity, majority voting works perfectly well.[7] Non-simple agendas may or may not have some additional properties. For example, they may or may not have a minimally inconsistent subset with the special property that, by negating some even number of propositions in it, it becomes consistent; call an agenda of this kind *even-number-negatable*.[8] The agendas in our two examples above – the court and government examples – have both of these properties. In each case, the agenda has a minimally inconsistent subset of three propositions, and in that same subset one can find two propositions (i.e., an even number) whose negation renders the subset consistent: p and $\neg q$ in the case of $\{p, p \rightarrow q, \neg q\}$ in the government example, and p and q in the case of $\{p, q, \neg r\}$ in the court example, subject to the constraint $r \leftrightarrow (p \wedge q)$. Both properties – non-simplicity and even-number-negatability – are relatively undemanding, and only very restrictive kinds of agendas violate them.[9] Yet, going back to the original conditions of Theorem 1, with anonymity dropped, the following pair of results turns out to hold.

Theorem 2.

(a) If (and only if) the agenda is non-simple and even-number-negatable, every aggregation rule satisfying universal domain, collective rationality and systematicity is a dictatorship or inverse dictatorship (Dietrich and List 2007a).

(b) If (and only if) the agenda is non-simple (whether or not it is even-number-negatable), every aggregation rule satisfying the same conditions and in addition monotonicity is a dictatorship (Nehring and Puppe 2002, forthcoming).

Monotonicity the following natural requirement, also exemplified by majority voting:

Monotonicity. If a proposition $p \in X$ is collectively accepted for a given profile $(J_1, ..., J_n)$ and we consider another profile $(J_1, ..., J_i', ..., J_n)$ in which an additional individual i accepts p (i.e., $p \in J_i'$ whereas $p \notin J_i$) while all other individuals' judgment sets are as before, then p is also collectively accepted for the second profile.

If we introduce an additional restriction on the agenda, then a similar pair of results holds with systematicity weakened to independence and an additional responsiveness condition of *unanimity preservation*. Call an agenda *totally blocked* (also called *path-connected*) if any proposition contained in it can be 'reached' from any other via a

[7] The majority judgments are then consistent and in the absence of ties also complete.

[8] This property was introduced by Dietrich (2007a) and Dietrich and List (2007a). A logically equivalent property is the algebraic property of *non-affineness* introduced by Dokow and Holzman (forthcoming), which requires that the admissible set of 0/1-truth-evaluations across the propositions in X should not be an affine subspace of $\{0, 1\}^{\frac{|X|}{2}}$, where $\frac{|X|}{2}$ is the total number of proposition-negation pairs in X.

[9] Non-simplicity is only violated if the logical interconnections in X do not go beyond pairs of propositions. Even-number-negatability is only violated if X is isomorphic to a set of propositions in standard propositional logic whose only logical connectives are $\neg$ and $\leftrightarrow$ (Dokow and Holzman forthcoming).

sequence of pairwise logical entailments conditional on other propositions in the agenda (Nehring and Puppe 2002, forthcoming). Formally, $p \in X$ *conditionally entails* $q \in X$, written $p \vdash^* q$, if there exists a subset $Y \subseteq X$ consistent with each of p and $\neg q$ such that $\{p\} \cup Y \vdash q$. The agenda X is *totally blocked* if, for any $p, q \in X$, there exists a sequence of propositions p_1,...,$p_k \in X$ such that p=$p_1 \vdash^* p_2 \vdash^*$ ... $\vdash^* p_k$=q. Any agenda with this property is automatically non-simple, as defined above. The following pair of results holds.

Theorem 3.
(a) If (and only if) the agenda is totally blocked and even-number-negatable, every aggregation rule satisfying universal domain, collective rationality, independence and unanimity preservation is a dictatorship (Dokow and Holzman forthcoming; the 'if'-part was also proved in Dietrich and List 2007a).
(b) If (and only if) the agenda is totally blocked (whether or not it is even-number-negatable), every aggregation rule satisfying the same conditions and in addition monotonicity is a dictatorship (Nehring and Puppe 2002, forthcoming).

Unanimity preservation can be formally stated as follows; again, it is satisfied by majority voting.

Unanimity preservation. For any admissible unanimous profile $(J, ..., J)$, $F(J, ..., J) = J$.

Table 3 summarizes the four agenda characterization results stated in this subsection.

Class of agendas	**Input**	**Output**	**Resp'ness**	**Resulting agg. rules**
Non-simple Even-numb.-neg.	Univ. domain	Coll. rationality	Systematicity	Dictatorships Inv. dict'ships
(Theorem 2a)				
Non-simple	Univ. domain	Coll. rationality	Systematicity Monotonicity	Dictatorships
(Theorem 2b)				
Totally blocked Even-numb.-neg.	Univ. domain	Coll. rationality	Independence Unan. preserv.	Dictatorships
(Theorem 3a)				
Totally blocked	Univ. domain	Coll. rationality	Independence Monotonicity Unan. preserv.	Dictatorships
(Theorem 3b)				

Table 3: Agenda characterization results

For each row of the table, the following two things are true: first, if the agenda has the property indicated in the left-most column, every aggregation rule satisfying the specified input, output and responsiveness conditions is of the kind stated in the right-most column; and second, if the agenda violates the property in the left-most column, there exist aggregation rules other than those stated in the right-most column which still satisfy the specified conditions.

In the next subsection, I give a pedagogical sketch proof of the impossibility ('if') part of Theorem 3a, from which the impossibility parts of Theorems 2a,b and 3b can, in turn, be derived. The proof to be presented draws on Dietrich and List (2007a). Less technically inclined readers may skip that subsection without losing the general thread of the discussion.

4.4 A sketch proof

Suppose X is totally blocked and even-number-negatable, and F satisfies the conditions of Theorem 3a, i.e., universal domain, collective rationality, independence, and unanimity preservation. We want to show that F must be a dictatorship of one individual.

Since F satisfies independence, the question of whether any proposition $p \in X$ is collectively accepted for any given profile depends only on the individual judgments on p, not on individual judgments on other propositions. In particular, some combinations of individual judgments on p lead to the collective acceptance of p, others to its rejection. We call a set of individuals $C \subseteq N$ a *winning coalition* for p if every profile in which all the individuals in C accept p while all others reject p leads to the collective acceptance of p. We write $\mathcal{C}_p$ to denote the set of all winning coalitions for p. For each $p \in X$, the set $\mathcal{C}_p$ fully encodes the functional relationship between individual and collective judgments on p. Thus the aggregation rule F can be represented as follows. For any admissible profile $(J_1, ..., J_n)$,

$$F(J_1, ..., J_n) = \{p \in X : \{i \in N : p \in J_i\} \in \mathcal{C}_p\},$$

i.e., the set of collectively accepted propositions consists of every proposition that is accepted by a winning coalition for it. Using this insight, the argument that, under the conditions of Theorem 3a, F is a dictatorship of one individual proceeds in five steps, which successively constrain the properties of the sets of winning coalitions $\mathcal{C}_p$ across the propositions in X.

> **Claim 1.** If a proposition $p \in X$ conditionally entails another proposition $q \in X$, then every winning coalition for p is also a winning coalition for q. Thus, if $p \vdash^* q$, then $\mathcal{C}_p \subseteq \mathcal{C}_q$.

To show this, suppose $p \vdash^* q$ and $C \subseteq N$ is a winning coalition for p. Since $p \vdash^* q$, there exists a subset $Y \subseteq X$ consistent with each of p and $\neg q$ such that $\{p\} \cup Y \vdash q$. Now one can construct an admissible profile within the universal domain in which the individuals in C accept all propositions in $\{p, q\} \cup Y$ (which can be verified to be consistent) and the individuals in $N \backslash C$ accept all propositions in $\{\neg p, \neg q\} \cup Y$ (which can also be verified to be consistent). Since all individuals in N accept all the propositions in Y, and N is a winning coalition for each such proposition (by unanimity preservation, together with independence), all propositions in Y are collectively accepted. Since C is a winning coalition for p, p is collectively accepted as well. But since $\{p\} \cup Y \vdash q$, collective rationality requires q to be collectively accepted too (if it is not, then by completeness $\neg q$ must be accepted and the collective judgments will be inconsistent). Therefore C must be a winning coalition for q as well.

Claim 2. The set of winning coalitions $\mathcal{C}_p$ is the same for all propositions $p \in X$, and thus F can be represented in terms of a single set of winning coalitions $\mathcal{C}$.

Given claim 1, claim 2 follows immediately from the total blockedness of X. For any $p, q \in X$, we can find a sequence of propositions $p_1,...,p_k \in X$ such that $p{=}p_1 \vdash^* p_2 \vdash^* ... \vdash^* p_k{=}q$, and hence $\mathcal{C}_p{=}\mathcal{C}_{p_1} \subseteq \mathcal{C}_{p_2} ... \subseteq \mathcal{C}_{p_k}{=}\mathcal{C}_q$, by claim 1. But we can also find a similar sequence that takes us from q to p, and hence $\mathcal{C}_q \subseteq \mathcal{C}_p$. Therefore $\mathcal{C}_p = \mathcal{C}_q$. From now on, we can write $\mathcal{C}$ to denote the universal set of winning coalitions for all propositions, dropping the proposition-specific subscript p.

Claim 3. If C is a winning coalition, then so is any superset of C. Thus $\mathcal{C}$ is superset-closed.

Let $C \in \mathcal{C}$, and consider any $C^* \supseteq C$. Since X is even-number-negatable, it has a minimally inconsistent subset Y with the special property that there exists a subset $Z \subseteq Y$ of even size such that $(Y \backslash Z) \cup \{\neg p : p \in Z\}$ is consistent. Without loss of generality, it can be assumed that Z has precisely two members (for reasons of space, I omit the argument why this is no loss of generality). So let $Z = \{p, q\}$. Since the sets

$$\begin{aligned}&(Y \backslash \{q\}) \cup \{\neg q\},\\ &(Y \backslash \{p, q\}) \cup \{\neg p, \neg q\},\\ &(Y \backslash \{p\}) \cup \{\neg p\}\end{aligned}$$

are each consistent (the first and third by the minimal inconsistency of Y, and the second by even-number-negatability), one can construct an admissible profile in the universal domain in which the individuals in C accept all the propositions in the first set, the individuals in $C^* \backslash C$ accept all the propositions in the second set, and the rest, i.e., those in $N \backslash C^*$, accept all the propositions in the third set. The restriction of this profile to the relevant propositions is represented in Table 4.

	p	q	all propositions in $Y \backslash \{p, q\}$
Group 1: C	true	false	true
Group 2: $C^* \backslash C$	false	false	true
Group 3: $N \backslash C^*$	false	true	true

Table 4: A profile (displayed only for the relevant propositions)

Since p is accepted precisely by the individuals in C and C is a winning coalition, p is collectively accepted. Further, since the propositions in $Y \backslash \{p, q\}$ are all accepted by all individuals and N is a winning coalition, they are all collectively accepted. But now, since Y is minimally inconsistent, collective rationality implies that $\neg q$ must be collectively accepted too (if it is not, then by completeness q must be accepted and the collective judgments will be inconsistent). But since $\neg q$ is accepted precisely by the individuals in C^* (i.e., those in $C \cup (C^* \backslash C)$), this set must be a winning coalition, i.e., in $\mathcal{C}$.

Claim 4. The intersection of any two winning coalitions C_1 and C_2 is also a winning coalition. Thus $\mathcal{C}$ is intersection-closed.

Let $C_1, C_2 \in \mathcal{C}$. By the non-simplicity of the agenda (which can be shown to be implied by its total blockedness), there exists a minimally inconsistent subset $Y \subseteq X$ containing three or more propositions. Let p, q, r be three distinct propositions in Y. Since the sets

$$(Y\backslash\{r\}) \cup \{\neg r\},$$
$$(Y\backslash\{q\}) \cup \{\neg q\},$$
$$(Y\backslash\{p\}) \cup \{\neg p\}$$

are each consistent, one can construct an admissible profile in the universal domain in which the individuals in $C_1 \cap C_2$ accept all the propositions in the first set, the individuals in $C_1 \backslash C_2$ accept all the propositions in the second set, and all others, i.e., those in $N \backslash C_1$, accept all the propositions in the third set. The restriction of the profile to the relevant propositions is represented in Table 5.

	p	q	r	all propositions in $Y\backslash\{p,q,r\}$
Group 1: $C_1 \cap C_2$	true	true	false	true
Group 2: $C_1 \backslash C_2$	true	false	true	true
Group 3: $N \backslash C_1$ ($\supseteq C_2 \backslash C_1$)	false	true	true	true

Table 5: A profile (displayed only for the relevant propositions)

Since p is accepted precisely by the individuals in C_1 (everyone in $(C_1 \cap C_2) \cup (C_1 \backslash C_2)$) and C_1 is a winning coalition, p is collectively accepted. Further, since q is accepted by the individuals in a superset of the winning coalition C_2 (namely everyone in $(N \backslash C_1) \cup (C_1 \cap C_2)$) and supersets of winning coalitions are themselves winning coalitions (by claim 3), q is also collectively accepted. Finally, since the propositions in $Y \backslash \{p, q, r\}$ are accepted by all individuals and N is a winning coalition, they are also collectively accepted. But now, since Y is minimally inconsistent, collective rationality will be violated unless $\neg r$ is collectively accepted (if it is not, then by completeness r must be accepted and the collective judgments will be inconsistent). But since $\neg r$ is accepted precisely by the individuals in $C_1 \cap C_2$, this set must be a winning coalition, i.e., in $\mathcal{C}$.

Claim 5. There exists a single individual i (a 'dictator') such that, for any $C \subseteq N$, $C \in \mathcal{C}$ if and only if $i \in C$. Thus F is a dictatorship.

By claim 4, the intersection of all winning coalitions is a winning coalition. Moreover, this intersection must be non-empty since, as noted, N is a winning coalition and thus the empty set cannot be a winning coalition (otherwise we would run into violations of collective rationality). Hence there exists an individual i who is a member of every winning coalition $C \in \mathcal{C}$. Now suppose, for a contradiction, that there exists a set $C \subseteq N$ that contains individual i but is not a winning coalition. In order to avoid violations of the completeness part of collective rationality, the complement of C, i.e., $N \backslash C$, must then be a winning coalition (otherwise the simultaneous rejection of a proposition and its negation becomes possible). But this complement does not contain individual i, which contradicts the earlier observation that individual i is a member of every winning coalition. This completes the proof of the impossibility part of Theorem 3a. ■

For the possibility ('only if') part of Theorem 3a, a proof of which I omit here, we need to show that, for any agenda X that violates total blockedness or even-number-negatability, one can construct an aggregation rule F satisfying the specified conditions. The reader is referred to Dokow and Holzman (forthcoming) and Nehring and Puppe (2002, forthcoming) for explicit constructions.

It remains to explain how we can derive proofs of the impossibility ('if') parts of Theorems 2a,b and 3b from the present proof. Since the proof I have sketched is quite 'modular', each of these other proofs can be obtained simply by dropping a suitable module. Let me go through each case.

- Theorem 3b imposes the additional condition of monotonicity on F. Claim 3 therefore follows immediately from the monotonicity of F and does not need to be derived from other assumptions. Consequently, it is no longer necessary to invoke the agenda condition of even-number-negatability, which is used only in the argument for claim 3. The proof now goes through under the agenda condition of total blockedness alone.
- Theorem 2a imposes the condition of systematicity, not just independence, on F. Claim 2 therefore follows immediately from the neutrality part of systematicity, without any further argument, and claim 1 becomes redundant. In consequence, the agenda condition of total blockedness is no longer needed and can be weakened to non-simplicity alone. The rest of the proof still works as before.
- Theorem 2b imposes both systematicity and monotonicity on F. As a result, the derivations of claims 1, 2 and 3 – and with them the agenda conditions of total blockedness and even-number-negatability – can be dropped. The rest of the proof goes through under the agenda condition of non-simplicity alone.

Later, I revisit the proof one more time, in connection with an impossibility result that relaxes the condition of collective rationality, showing that the basic argument can also be adjusted to apply to that case.

5 Avoiding the impossibility

The theorems reviewed in the last section show that:

> **Remark 4.** *If* (i) we deal with decision problems in which the agenda exhibits some of the identified properties *and* (ii) we consider the specified input, output and responsiveness conditions to be indispensable requirements of democratic aggregation, *then* judgment aggregation problems have no non-degenerate solutions.

To avoid such a negative implication, we must therefore deny either (i) or (ii). Unless we can somehow avoid non-trivial decision problems altogether, denying (i) does not seem to be a viable option. Therefore we must deny (ii). So what options do we have? For each of the three types of conditions on an aggregation rule that have been introduced, we can ask whether a suitable relaxation would enable us to avoid the impossibility.

5.1 Relaxing the input conditions

The impossibility theorems reviewed in the previous section all impose the condition of universal domain on the aggregation rule, by which any possible profile of consistent and complete individual judgment sets on the propositions in X is deemed admissible as input to the aggregation. At first sight, this condition seems very reasonable, since we want the aggregation rule to cope with all possible inputs that may be submitted to it. However, different groups may exhibit different levels of pluralism, and in some groups there may be more agreement between the members' judgments than in others. Expert panels or ideologically well structured societies may be more homogeneous than large and internally diverse electorates, for example. Thus the profiles of individual judgment sets leading to collective inconsistencies under plausible aggregation rules such as majority voting may be more likely to occur in heterogeneous groups than in more homogeneous ones. Can we say something about the kind of homogeneity that is required for the avoidance of majority inconsistencies – and by implication for the avoidance of the more general impossibility results presented?

It turns out that there exist several combinatorial conditions with the property that, on the restricted domain of profiles of individual judgment sets satisfying those conditions, majority voting generates consistent and (absent ties) complete individual judgment sets. Of course, majority voting also satisfies the various responsiveness conditions introduced in the last section. Given space constraints, I can here discuss only two illustrative such conditions: a very simple one and a very general one.

The first condition, called *unidimensional alignment* (List 2003), is similar in spirit, but not equivalent, to the classic condition of *single-peakedness* in the theory of preference aggregation, which was introduced by Black (1948). (Single-peakedness is a constraint on profiles of preference orderings rather than on profiles of judgment sets.) A profile $(J_1, ..., J_n)$ is *unidimensionally aligned* if the individuals in N can be aligned from left to right such that, for every proposition $p \in X$, the individuals accepting p (i.e., those in $\{i \in N : p \in J_i\}$) are either all to the left, or all to the right, of those rejecting p (i.e., those in $\{i \in N : p \notin J_i\}$), as illustrated in Table 6.

	Ind. 1	Ind. 2	Ind. 3	Ind. 4	Ind. 5
p	True	False	False	False	False
$p \to q$	False	True	True	True	True
q	False	False	False	True	True

Table 6: A unidimensionally aligned profile of individual judgment sets

The relevant left-right alignment of the individuals may be interpreted as capturing their position on some cognitive or ideological dimension (e.g., from socio-economic left to right, or from urban to rural, or from secular to religious, or from risk-averse to risk-taking etc.), but what matters from the perspective of achieving majority consistency is not the semantic interpretation of the alignment but rather the combinatorial constraint it imposes on individual judgments.

Why is unidimensional alignment sufficient for consistent majority judgments? Since the individuals accepting each proposition are opposite those rejecting it on the given left-right alignment, a proposition cannot be accepted by a majority unless it is accepted by the middle individual on that alignment[10] – individual 3 in the example of Table 6. In particular, the majority judgments must coincide with the middle individual's judgments.[11] Hence, assuming that the middle individual holds consistent judgments, the resulting majority judgments will be consistent too.[12] When restricted to the domain of unidimensionally aligned profiles of consistent and complete individual judgment sets, majority voting therefore satisfies all the conditions introduced in the last section, except of course universal domain.

However, while unidimensional alignment is sufficient for majority consistency, it is not necessary. A necessary *and* sufficient condition is the following (Dietrich and List 2007c). A profile $(J_1, ..., J_n)$ is called *majority consistent* if every minimally inconsistent subset $Y \subseteq X$ contains at least one proposition $p \in Y$ that is *not* accepted by a majority, i.e., for which $|\{i \in N : p \in J_i\}| \leq \frac{n}{2}$. It is easy to see that this is enough to ensure consistent majority judgments. Suppose the set of propositions accepted by a majority, i.e., $J = F(J_1, ..., J_n)$ (where F is majority voting), is inconsistent. Then J must have at least one minimally inconsistent subset $Y \subseteq J$. But if the profile $(J_1, ..., J_n)$ satisfies the combinatorial condition just defined, then at least one proposition $p \in Y$ is not accepted by a majority, contradicting the assumption that all propositions in Y, being a subset of J, are majority-accepted.

An important special case of this combinatorial condition is the condition of *value restriction* (Dietrich and List 2007c), which can be shown to generalize the equally named classic condition in the context of preference aggregation (Sen 1966). A profile $(J_1, ..., J_n)$ is called *value-restricted* if every minimally inconsistent subset $Y \subseteq X$ contains a pair of propositions p, q not jointly accepted by *any* individual, i.e., for all $i \in N$, $\{p, q\} \nsubseteq J_i$. Again, this is enough to rule out that any minimally inconsistent set $Y \subseteq X$ can be majority-accepted: if it were, then each of the propositions $p, q \in Y$ from the definition of value restriction would be majority-accepted and thus at least one individual $i \in N$ would accept both, contradicting value restriction. Several other domain restriction conditions are discussed in Dietrich and List (2007c).

How plausible is the strategy of avoiding the impossibility of non-degenerate judgment aggregation via restricting the domain of admissible inputs to the aggregation rule? The answer to this question depends on the group, context and decision problem at stake. As already noted, different groups exhibit different levels of pluralism, and it is an empirical question whether or not any of the identified combinatorial conditions are met by the empirically occurring profiles of individual judgment sets in any given real-world case. Some groups may be naturally homogeneous and lined up along a one-dimensional ideological or cognitive spectrum. Consider, for example, societies with a strong tradition of a conventional ideological left-right polarization. Other societies or groups may not exhibit such a structure, and yet through group deliberation or other forms of communication they may be able to achieve sufficiently 'cohesive' individual judgments, which meet conditions such as unidimensional alignment or value restriction. In discussions of the relationship between social choice theory and the theory of deliberative democracy, the existence of mechanisms along these lines has been suggested (Miller 1992, Knight and Johnson 1994, Dryzek and List 2003). However, the present escape route from the

[10] Or the middle two individuals, if n is even.

[11] Or the intersection of the judgments of the middle two individuals, if n is even.

[12] Similarly, if n is even, the intersection of the individually consistent judgment sets of the middle two individuals is still a consistent set of propositions.

impossibility is certainly no 'one size fits all' solution.

5.2 Relaxing the output conditions

Like the input condition of universal domain, the output condition of collective rationality occurs in all the impossibility theorems reviewed above. Again the condition seems *prima facie* reasonable. First of all, the requirement of consistent collective judgments is important not only from a pragmatic perspective – after all, inconsistent judgments fail to be action-guiding when it comes to making concrete decisions – but also from a more fundamental philosophical one. As argued by Pettit (2001), collective consistency is essential for the contestability and justifiability of collective decisions (for critical discussions of this point, see also Kornhauser and Sager 2004 and List 2006). Secondly, the requirement of complete collective judgments, too, is pragmatically important. In many cases – and perhaps even by definition – the agenda consists precisely of those propositions that require actual adjudication; and if this is so, the formation of complete collective judgments on them is essential.

Nonetheless, the case for collective consistency is arguably stronger than that for collective completeness. There are now several papers in the literature that discuss relaxations of completeness (e.g., List and Pettit 2002; Gärdenfors 2006; Dietrich and List 2007b,d, 2008a; Dokow and Holzman 2006). Gärdenfors (2006), for instance, criticizes completeness as a 'strong and unnatural assumption'. However, not every relaxation of completeness is enough to avoid the impossibility of non-degenerate judgment aggregation. As shown by Gärdenfors (2006) for a particular class of agendas (so-called *atomless* agendas) and subsequently generalized by Dietrich and List (2008a) and Dokow and Holzman (2006), if the collective completeness requirement is weakened to a *deductive closure* requirement, then the other conditions reviewed above restrict the possible aggregation rules to so-called *oligarchic* ones. *Deductive closure* requires that all propositions in X that are logically entailed by other accepted propositions be also accepted, i.e., if $J \vdash p$, then $p \in J$. An aggregation rule is *oligarchic* if there exists an antecedently fixed non-empty subset $M \subseteq N$ – the *oligarchs* – such that the collective judgment set is always the intersection of the individual judgment sets of the oligarchs, formally $F(J_1, ..., J_n) = \bigcap_{i \in M} J_i$. A dictatorial aggregation rule is the limiting case in which the set of oligarchs M is singleton. In fact, a table very similar to Table 3 above can be derived in which the output condition is relaxed to the conjunction of consistency and deductive closure and the right-most column is extended to the class of oligarchic aggregation rules (Dietrich and List 2008a).

(Readers who wish to go back to the proof of Theorem 3a above can verify that if collective rationality is weakened to collective consistency and deductive closure, with all other conditions remaining as before, then all claims except the last one still go through. The weakened rationality requirement is strong enough for the arguments for claims 1 to 4 to continue to work, sometimes with minor adjustments. Claim 5, however, must be modified. While the intersection of all winning coalitions continues to be winning and non-empty, it need not be singleton any longer. Instead, it is a non-empty but possibly non-singleton set $M \subseteq N$ such that, for any $C \subseteq N$, $C \in \mathcal{C}$ if and only if $M \subseteq C$. This establishes the impossibility part of the main theorems in Dietrich and List 2008a and Dokow and Holzman 2006. As before, the proof can be adjusted to derive the oligarchy analogues of the impossibility parts of Theorems 2a,b and 3b as well.)

If collective rationality is weakened further, namely to consistency alone, more promising possibilities open up. Groups may then use supermajority rules according to which any proposition on the agenda is collectively accepted if and only if it is accepted by a certain supermajority of individuals, such as more than two thirds, three quarters, or four fifths of them. If the supermajority threshold $q \in (\frac{1}{2}, 1]$ is greater than $\frac{k-1}{k}$, where k is the size of the largest minimally inconsistent subset of the agenda, such rules always produce consistent (but not generally deductively closed) collective judgments (Dietrich and List 2007b, extending List and Pettit 2002). To see this, suppose, for a contradiction, that an inconsistent set of propositions is collectively accepted under such a rule. Then all the propositions in at least one minimally inconsistent subset $Y \subseteq X$, which by assumption is of size k or smaller, are each accepted by a supermajority of more than $\frac{k-1}{k}$ of the individuals. But any k or fewer supermajorities of that size must have at least one individual in common – just as two simple majorities, or three majorities of more than two thirds, must have at least one individual in common – and this individual must accept all k propositions at once. Given the inconsistency of Y, however, this contradicts the consistency of the individual's judgments and thus constitutes a violation of universal domain. In the court and government examples above, we have $k = 3$, and thus a supermajority threshold above $\frac{2}{3}$ would be sufficient for collective consistency. Supermajority rules, of course, satisfy all the other (input and responsiveness) conditions that I have reviewed.

Groups with a strongly consensual culture, such as the UN Security Council or the EU Council of Ministers, may well adopt this supermajoritarian approach to solving judgment aggregation problems. The price they have to pay for avoiding the impossibility of non-degenerate judgment aggregation in this manner is the risk of stalemate. Small minorities are able to veto judgments on any propositions. Furthermore, when *both* individual *and* collective judgment sets are only required to be consistent but neither complete nor deductively closed, a recent impossibility theorem suggests that an asymmetry in the criteria for accepting and for rejecting propositions is a necessary condition for avoiding dictatorial aggregation rules (Dietrich and List 2007d). As in the case of the earlier escape route – via relaxing universal domain – the present route is no 'one size fits all' solution to the problem of judgment aggregation.

5.3 Relaxing the responsiveness conditions

Arguably, the most compelling escape-route from the impossibility of non-degenerate judgment aggregation opens up when we relax some of the responsiveness conditions used in the impossibility theorems. The key condition here is independence, i.e., the first part of the systematicity condition, which requires that the collective judgment on each proposition on the agenda depend only on individual judgments on that proposition, not on individual judgments on other propositions. The second part of systematicity, requiring that the criterion for determining the collective judgment on each proposition be the same across propositions, is already absent from several of the impossibility results, such as the two parts of Theorem 3, and relaxing it alone is insufficient for avoiding an impossibility result in general (especially when the agenda is above a certain level of complexity).

If we give up independence, however, several promising aggregation rules become possible. The simplest example is the premise-based rule, which I have already introduced above. This rule was discussed, originally under the name *issue-by-issue voting*, by Kornhauser and Sager (1986) and Kornhauser (1992), and later by Pettit (2001), List and Pettit (2002), Chapman (2002), Bovens and Rabinowicz (2006), Dietrich (2006) and many others. As noted, the premise-based rule involves designating a subset $Y \subseteq X$ as a set of *premises* and generating collective judgments by taking majority votes on all premises and then deriving the judgments on all other propositions (*conclusions*) from these majority judgments on the premises. By construction, the consistency of the resulting collective judgments is guaranteed, provided the premises are logically independent from each other. If these premises further constitute a *logical basis* for the entire agenda – i.e., they are not only logically independent but any assignment of truth-values to them also settles the truth-values of all other propositions – then the premise-based rule also ensures collective completeness.[13] (The conclusion-based rule, by contrast, violates completeness, in so far as it only ever generates collective judgments on the conclusion(s), by taking majority votes on them alone.)

The premise-based rule, in turn, is a special case of a *sequential priority rule* (List 2004). To define such an aggregation rule, we must specify a particular order of priority among the propositions in X such that earlier propositions in that order are interpreted as epistemically (or otherwise) prior to later ones. For each given profile $(J_1, ..., J_n)$, the propositions in X are then considered one-by-one in the specified order and the collective judgment on $p \in X$ is formed as follows. If the majority judgment on p is consistent with the collective judgments on propositions considered earlier, then that majority judgment on p becomes the collective judgment on p; but if the majority judgment on p is inconsistent with those earlier judgments on other propositions, then the collective judgment on p is determined by the implications of those earlier judgments. In the example of Table 2 above, the multi-member government might consider the propositions in the order p, $p \to q$, q (with negations interspersed) and then accept p and $p \to q$ by majority voting while accepting q by logical inference. The collective judgment set under such an aggregation rule is dependent on the specified order of priority among the propositions. This property of *path dependence* can be seen as a virtue or as a vice, depending on the perspective one takes. On the one hand, it appears to do justice to the fact that propositions can differ in their status (consider, for example, constitutional propositions versus propositions of ordinary law), as emphasized by Pettit (2001) and Chapman (2002). But on the other hand, it makes collective judgments manipulable by an agenda setter who can influence the order in which propositions are considered (List 2004), which in turn echoes a much-discussed worry in social choice theory (e.g., Riker 1982).

Another class of aggregation rules that give up independence – the class of *distance-based rules* – was introduced by Pigozzi (2006), drawing on related work in the area of belief merging in computer science (Konieczny and Pino Pérez 2002). Unlike premise-based or sequential priority rules, these rules are not based on the idea of prioritizing some propositions over others. Instead, they are based on a distance metric between judgment

[13] A first general formulation of a premise-based rule in terms of a subset $Y \subseteq X$ interpreted as the set of premises was given in List and Pettit (2002). Furthermore, as shown by Dietrich (2006), premise-based rules can be axiomatically characterized in terms of the key condition of *independence restricted to* Y, where Y is the premise-set. In some cases, an impossibility result reoccurs when the preservation of unanimous individual judgments on non-premises is required, as shown for certain agendas by Mongin's (2008) theorem mentioned in the previous section. For recent extensions, see Dietrich and Mongin (2007).

sets. We can define the *distance* between any two judgment sets $J, J' \subseteq X$ for instance by counting the number of propositions on the agenda on which they disagree, i.e., $d(J, J') = |\{p \in X : p \in J \nLeftrightarrow p \in J'\}|$. (This particular metric d is called the *Hamming distance.*) A distance-based aggregation rule now assigns to each profile $(J_1, ..., J_n)$ a consistent and complete collective judgment set J that minimizes the sum-total distance from the individual judgment sets, $\sum_{i \in N} d(J, J_i)$, with some additional stipulation for dealing with ties. Distance-based aggregation rules have a number of interesting properties. They capture the idea of reaching a compromise between different individuals' judgment sets. Further, although they give up independence, they can preserve the spirit of neutrality across propositions if they are defined in terms of a distance metric such as the Hamming distance that treats all propositions on the agenda equally.

What is the cost of violating independence? Arguably, the greatest cost is manipulability of the aggregation rule by the submission of insincere individual judgments (Dietrich and List 2007e). Call an aggregation rule *manipulable* if there exists at least one admissible profile $(J_1, ..., J_n)$ such that the following is true for at least one individual $i \in N$ and at least one proposition $p \in X$: (i) if individual i submits his or her genuine judgment set J_i, then the collective judgment on p differs from the individual's genuine judgment on p, i.e., $p \in F(J_1, ..., J_n) \nLeftrightarrow p \in J_i$; (ii) if individual i submits a strategically adjusted judgment set J_i', then the collective judgment on p coincides with the individual's genuine judgment on p (where other individuals' judgment sets remain equal), i.e., $p \in F(J_1, ..., J_i', ...J_n) \Leftrightarrow p \in J_i$. If an aggregation rule is manipulable in this sense, then individuals may have incentives to misrepresent their judgments.[14] To illustrate, if the court in the example of Table 1 were to use the premise-based rule, sincere voting among the judges would lead to a 'liable' verdict, as we have seen. However, if judge 3 were sufficiently strongly opposed to this outcome, he or she could strategically manipulate the outcome by pretending to believe that q is false, contrary to his or her sincere judgment; the result would be the majority rejection of q, and consequently a 'not liable' verdict. It can be shown that an aggregation rule is *non*-manipulable if and only if it satisfies the conditions of independence and monotonicity (Dietrich and List 2007e; for closely related results in a more classic social-choice-theoretic framework, see Nehring and Puppe 2007). Assuming that, other things being equal, the relaxation of independence is the most promising way to make non-degenerate judgment aggregation possible, the impossibility theorems reviewed above can therefore be seen as pointing to a trade-off between degeneracy of judgment aggregation on the one hand (most notably, in the form of dictatorship) and its potential manipulability on the other. As in other branches of social choice theory, a perfect aggregation rule does not exist.

6 The relationship to other aggregation problems

Before concluding, it is useful to consider the relationship between the theory of judgment aggregation and other branches of aggregation theory. I here focus on three related aggregation problems: preference aggregation, abstract aggregation and probability aggregation.

[14] The precise relationship between *opportunities* and *incentives* for manipulation is discussed in Dietrich and List (2007e).

6.1 Preference aggregation

The theory of preference aggregation in the long and established tradition of Condorcet and Arrow addresses the following question: How can a group of individuals arrive at a collective preference ordering on some set of alternatives on the basis of the group members' individual preference orderings on them? Condorcet's classic paradox illustrates some of the challenges raised by this problem. Consider a group of individuals seeking to form collective preferences over three alternatives, x, y and z, where the first individual prefers x to y to z, the second y to z to x, and the third z to x to y. In this case, majority voting over pairs of alternatives fails to yield a rational collective preference ordering: there are majorities for x over y, for y over z, and yet for z over x – a *preference cycle*. Arrow's theorem (1951/1963) generalizes this observation by showing that, when there are three or more alternatives, the only aggregation rules that guarantee the avoidance of cycles and satisfy some other minimal conditions are dictatorial ones. Condorcet's paradox and Arrow's theorem have inspired a huge literature on axiomatic social choice theory, a review of which is entirely beyond the scope of this paper.

How is the theory of preference aggregation related to the theory of judgment aggregation? It turns out that preference aggregation problems can be formally represented within the model of judgment aggregation, as presented here. The idea is that preference orderings can be represented as sets of accepted preference ranking propositions of the form '*x is preferable to y*', '*y is preferable to z*', and so on.

To construct this representation formally (following Dietrich and List 2007, extending List and Pettit 2004), it is necessary to employ a specially devised predicate logic with two or more constants representing alternatives, denoted x, y, z and so on, and a two-place predicate '_*is preferable to*_'. To capture the standard rationality conditions on preferences (such as asymmetry, transitivity and connectedness), we define a set of propositions in our predicate logic to be *consistent* just in case this set is consistent relative to those rationality conditions. For example, the set {*x is preferable to y*, *y is preferable to z*} is consistent, while the set {*x is preferable to y*, *y is preferable to z*, *z is preferable to x*} – representing a preference cycle – is not. The agenda X is then defined as the set of all propositions of the form '*v is preferable to w*' and their negations, where v and w are distinct alternatives among x, y, z and so on. Now each consistent and complete judgment set on X uniquely represents a rational (i.e., asymmetric, transitive and connected) preference ordering. For instance, the judgment set {*x is preferable to y*, *y is preferable to z*, *x is preferable to z*} uniquely represents the preference ordering that ranks x above y above z. Furthermore, a judgment aggregation rule on X uniquely represents an Arrovian preference aggregation rule, i.e., a function from profiles of individual preference orderings to collective preference orderings.

Under this construction, Condorcet's paradox of cyclical majority preferences becomes a special case of the problem of majority inconsistency discussed in section 2 above. To see this, notice that the judgment sets of the three individuals in the example of Condorcet's paradox are as shown in Table 7. Given these individual judgments, the majority judgments are indeed inconsistent, as the set of propositions accepted by a majority is inconsistent relative to the rationality condition of transitivity.

More generally, when there are three or more alternatives, the agenda X, as just defined, has all the combinatorial properties introduced in the discussion of the impossibility theorems above (i.e., non-simplicity, even-number-negatability, and total blockedness /

	x is preferable to y	y is preferable to z	x is preferable to z
Individual 1 ($x \succ y \succ z$)	True	True	True
Individual 2 ($y \succ z \succ x$)	False	True	False
Individual 3 ($z \succ x \succ y$)	True	False	False
Majority	True	True	False

Table 7: Condorcet's paradox translated into jugdment aggregation

path-connectedness), and thus those theorems apply to the case of preference aggregation. In particular, the only aggregation rules satisfying universal domain, collective rationality, independence and unanimity preservation are dictatorships (Dietrich and List 2007, Dokow and Holzman forthcoming; for a similar result with the additional condition of monotonicity, see Nehring 2003). This is precisely Arrow's classic impossibility theorem for strict preferences: the conditions of universal domain and collective rationality correspond to Arrow's equally named conditions, independence corresponds to Arrow's independence of irrelevant alternatives, and unanimity preservation, finally, corresponds to Arrow's weak Pareto principle.

6.2 Abstract aggregation

The problem of judgment aggregation is closely related to the problem of abstract aggregation first formulated by Wilson (1975) (in the binary version discussed here) and later generalized by Rubinstein and Fishburn (1986) (in a non-binary version). In recent work, the problem has been discussed by Dokow and Holzman (forthcoming) and in a slightly different formulation (the *property space* formulation) by Nehring and Puppe (2002, forthcoming). As before, let me begin by stating the key question: How can a group of individuals arrive at a collective vector of yes/no evaluations over a set of binary issues on the basis of the group members' individual evaluations over them, subject to some feasibility constraints? Suppose there are multiple binary issues on which a positive (1) or negative (0) view is to be taken. An 'evaluation vector' over these issues is an assignment of 0s and 1s to them. Let $Z \subseteq \{0,1\}^k$ be the set of evaluation vectors deemed *feasible*, where k is the total number of issues. Now an *abstract aggregation rule* is a function that maps each profile of individual evaluation vectors in a given domain of feasible ones to a collective evaluation vector. To represent Kornhauser and Sager's court example in this model, we introduce three issues, corresponding to propositions p, q and r, and define the set of feasible evaluation vectors to be $Z = \{(0,0,0),(0,1,0),(1,0,0),(1,1,1)\}$, i.e., the set of 0/1 assignments that respect the doctrinal constraint whereby positive evaluations on the first two issues (corresponding to p and q) are necessary and sufficient for a positive evaluation on the third one (corresponding to r). More generally, a judgment aggregation problem can be represented in the abstract aggregation model by defining the set of feasible evaluation vectors to be the set of admissible truth-value assignments to the unnegated propositions on the agenda. The problem of majoritarian inconsistency then reemerges as a failure of issue-wise majority voting to preserve feasibility from the individual to the collective level.

As discussed in List and Puppe (2009), the model of abstract aggregation is informationally sparser than the logic-based model of judgment aggregation. To see that translating judgment aggregation problems into abstract ones involves some informational loss, notice that very different agendas (and thereby very different decision problems) can give rise to the same set of feasible evaluation vectors. For example, the set of feasible evaluation vectors resulting from the agenda containing p, $p \leftrightarrow q$, $p \wedge q$ (and negations), without any doctrinal constraint, coincides with that resulting from the agenda in the court example – namely Z as just defined – although syntactically and interpretationally those agendas are very different from each other.

The abstract aggregation model is arguably at its strongest when our primary interest lies in how the existence of non-degenerate aggregation rules depends on the nature of the feasibility constraints, as opposed to the particular syntactic structure or interpretation of the underlying propositions. Indeed, the agenda characterization theorems reviewed above have their intellectual origins in the literature on abstract aggregation (and here particularly in Nehring and Puppe's 2002 work as well as in Dokow and Holzman's forthcoming subsequent paper). When the logical formulation of a decision problem is to be made explicit, or when the rationality constraints on judgments (and their possible relaxations) are to be analyzed using logical concepts, on the other hand, the logic-based model of judgment aggregation seems more natural.

6.3 Probability aggregation

In the theory of probability aggregation, finally, the focus is not on making consistent acceptance/rejection judgments on the propositions of interest, but rather on arriving at a coherent probability assignment to them (e.g., McConway 1981, Genest and Zidek 1986, Mongin 1995). Thus the central question is: How can a group of individuals arrive at a collective probability assignment to a given set of propositions on the basis of the group members' individual probability assignments, while preserving probabilistic coherence (i.e., the satisfaction of the standard axioms of probability theory)? This problem is quite general. In a number of decision-making settings, the aim is not so much to come up with acceptance/rejection judgments on certain propositions but rather to arrive at probabilistic information about the degree of belief we are entitled to assign to them or the likelihood of the events they refer to.

Interestingly, the move from a binary to a probabilistic setting opens up some non-degenerate possibilities of aggregation that do not exist in the standard case of judgment aggregation. A key insight is that probabilistic coherence is preserved under linear averaging of probability assignments. In other words, if each individual coherently assigns probabilities to a given set of propositions X, then any weighted linear average of these probability assignments across individuals still constitutes an overall coherent probability assignment on X. Moreover, it is easy to see that this method of aggregation satisfies the analogues of all the input, output and responsiveness conditions introduced above: i.e., it accepts all possible profiles of coherent individual probability assignments as input, produces a coherent collective probability assignment as output and satisfies the analogues of systematicy and unanimity preservation; it also satisfies anonymity if all individuals are given equal weight in the averaging. A classic theorem by McConway (1981) shows that, if the set of propositions X on which probabilities are to be assigned is isomorphic to a Boolean algebra with more than four elements, linear averaging is uniquely characterized by an independence condition, a unanimity preservation condition as well as the analogues of universal domain and collective rationality. Recently, Dietrich and List (2008b) have obtained a generalization of this theorem for a much larger class of agendas (essentially, the analogue of non-simple agendas). A challenge for the future is to obtain even more general theorems that yield both standard results on judgment aggregation and interesting characterizations of salient probability aggregation methods as special cases (for some ideas on how to move towards a general theory of propositional attitude aggregation, see Dietrich and List 2009).

7 Concluding remarks

The aim of this paper has been to give a brief introductory review of the theory of judgment aggregation. My focus has been on central ideas and questions of the theory and a few illustrative results. Inevitably, many important results and promising research directions have been omitted (for surveys, see, for example, List and Puppe 2009 and a forthcoming special issue of the *Journal of Economic Theory* on 'Judgment aggregation'). In particular, the bulk of this paper has focused on judgment aggregation in accordance with a systematicity or independence condition that forces the aggregation to take place in a propositionwise manner. Arguably, some of the most interesting open questions in the theory of judgment aggregation concern the relaxation of this propositionwise restriction and the move towards other, potentially more 'holistic' notions of responsiveness. Without the restriction to propositionwise aggregation, the space of possibilities grows dramatically, and I have here reviewed only a few examples of aggregation rules that become possible, namely premise-based, sequential priority and distance-based ones.

To provide a more systematic perspective on those possibilities, Dietrich (2007b) has recently introduced a general condition of *independence of irrelevant information*, defined in terms of a relation of informational relevance between propositions. An aggregation rule satisfies this condition just in case the collective judgment on each proposition depends only on individual judgments on propositions that are deemed relevant to it. In the classical case of propositionwise aggregation, each proposition is deemed relevant only to itself. In the case of a premise-based rule, by contrast, premises are deemed relevant to conclusions, while each premise is only relevant to itself; and in the case of a sequential priority rule, the relevance relation is given by a linear order of priority among the propositions. Important future research questions concern the precise interplay between the logical structure of the agenda, the relevance relation and the conditions on aggregation rules in determining the space of possibilities. A key question is to what extent the quest for non-degenerate judgment aggregation rules requires us to move away from local, propositionwise aggregation to holistic aggregation in which the collective judgment on each proposition may depend on individual judgments on entire 'webs' of other relevant logically connected propositions.

A further research direction considers the idea of decisiveness rights in the context of judgment aggregation, following Sen's classic work (1970) on the liberal paradox. In judgment aggregation, it is particularly interesting to investigate the role of experts and the question of whether we can arrive at consistent collective judgments when giving different individuals different weights depending on their expertise on the propositions in question. Some existing impossibility results (Dietrich and List 2008c) highlight the difficulties that can result from such deference to experts, but many open questions remain.

Finally, as in other areas of social choice theory, there is much research to be done on the relationship between aggregative and deliberative modes of decision-making. In many realistic settings, decision-makers do not merely mechanically aggregate their votes or judgments, but they exchange and share information, communicate with each other and update their beliefs. Some authors have begun to consider possible connections between the theory of judgment aggregation and the theory of belief revision (Pettit 2006, List forthcoming, Dietrich 2008, Pivato 2008). But much of this terrain is still unexplored. My hope is that this review will contribute to stimulating further research.

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